Photonics in Semiconductors, Spring 2017

Exercise 6, 2.3.2017

1. Waveguide condition

Derive the waveguide condition for a slab waveguide (thickness 2a, refractive indices are n_{core} and $n_{cladding}$, incidence angle between layers is θ_m, phase change at reflection: ϕ_m):

$$\left[\frac{2\pi n_{core}\,(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

2. Waveguide modes

Consider a planar dielectric waveguide with a core thickness 20 μm, $n_{core} = 1.455$, $n_{cladding} = 1.440$, $\lambda = 900\ nm$. The expression for ϕ for a TE mode is:

$$\tan\left(\frac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_{cladding}}{n_{core}}\right)\right]^{1/2}}{\cos\theta_m}$$

Compute solution for θ_m (numerically / graphically) and plot the results.

3. Waveguide mode shapes

a) Show that the phase difference between rays 1 and 2 at C is $\Phi_m = \Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$ (see the figure). Hint: Use geometrical reasoning to derive Φ_m as a function of θ. Then use the waveguide condition to get rid of explicit θ-dependence.
b) Derive the field variation as a function of y. Hint: Express the interfering waves as: $E(y) = A\cos(\omega t) + A\cos(\omega t + \Phi_m(y))$.
c) Plot $E(y)$ for first three TE modes. Hint: use the phase difference formula from problem 2. Use the following parameter values: $a = 10$ μm, $\lambda = 1.3$ μm, $n_{core} = 1.455$ and $n_{cladding} = 1.440$.