# Photonics in Semiconductors, Spring 2017

## Exercise 1, 19.1.2017

### 1. Maxwell's equations and waves

a) Derive the the free space wave equation for electric and magnetic fields from Maxwell's equations.
b) Harmonic plane wave propagates in linear dielectric with homogenous $\epsilon$ and $\mu_r = 1$. Derive the expression for magnetic field (amplitude, phase and polarization), if the complex electric field is linearily polarized in x-direction:

$$\mathbf{E} = E_0 e^{-i(\omega t - kz)} \hat{x}$$

### 2. Reflectance and transmittance

Starting from the Fresnel equations (see e.g. Hecht):
a) Derive the expressions for the amplitude coefficients for normal incidence for both TM and TE waves.
b) Derive the expression for coefficients of transmittance and reflectance at normal incidence:

$$\begin{aligned} R &= \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 \\ T &= \frac{4 n_1 n_2}{(n_2 + n_1)^2} \end{aligned}$$

c) What is the percendate of reflected irradiance at a air-glass interface (normal incidence, $n_{air} = 1$,$n_{glass} = 1.5$)?

### 3. Total internal reflection

A ray of light is incident at a interface between two linear dielectric media ($n_1 = 1.45$, $n_2 = 1.33$, vacuum wavelength:$\lambda = 1.064\ \mu m$)
a) Calculate the minimum angle for total internal reflection
b) Calculate the phase change in reflection, when $\theta_i = 70°$
c) Calculate the penetration depth, when $\theta_i = 70°$

### 4. Antireflection Coatings

a) Consider three dielectric media, with flat and parallel boundaries with refractive indices $n_1$, $n_2$ and $n_3$. Show that for normal incidence the reflection coefficient between layer 1 and 2 in the same as between layers 2 and 3 if $n_2 = \sqrt{n_1 n_3}$.
b) What should be the refractive index and thickness of an antireflection coating designed to operate at $\lambda_0 = 1.064\ \mu$m on a BK-7 lens in air.

### 5. Gaussian beam

Estimate the divergence and Rayleigh range of Gaussian beam from a HeNe laser with 1 mm beam width at $z = 0$. After traversing 10 m through vacuum, what will the beam width be?