(530190) Methods in Single Molecule Biophysics, Fall 2011

Exercise 2: Thursday 16.11.2011 at 12:15 in room D116

1. a) Starting from the equation of motion for a microsphere in water (assume Stokes drag), derive the Lorenzian power spectrum that we would expect if the bead is trapped in a harmonic potential and driven by a random thermal force a constant power spectral density of $4\ k_B T\ \gamma$. Assume m = 0. How does the PSD behave at low and high frequencies? Plot an example of the PSD.
b) How does the power spectrum change, if the mass of the particle is taken into account? Plot the spectrum for different values of m and compare with the results from a).

2. (Double points) There is some experimental data available for a 1 µm trapped microsphere on the course webpage:
http://electronics.physics.helsinki.fi/wp-content/uploads/2011/08/trapdata.zip
This is a time-series of position data along one dimension for a microsphere. The data is collected at 200 kS/s so the time-interval between each point is 5 µs. (The values are raw output of a 16-bit AD-conversion with voltage range: ±10V).

i) Download the data, import it into MATLAB or some other data-analysis program of your choice. You can remove the average of the data from all data points to center it around zero.
ii) Calculate and plot the power spectral density in a log-log figure (**pwelch** in MATLAB).
iii) Fit your result derived in 1a) to the data and determine the roll-off frequency. You will need two fitting parameters: an overall constant that scales the curve, and the roll-off frequency. In MATLAB you can use the function **lsqcurvefit** to do the fitting.
iv) What is the roll-off frequency? What is the stiffness of the trap?
v) If you compute a histogram of the time-series and use the stiffness you calculated in step iv), what is the position sensitivity of the detector that has been used?

3. Consider a one-dimensional harmonic well containing a 1 µm diameter plastic sphere The stiffness (spring constant) of the potential is k=1 pN/nm. What happens when the center of the harmonic well is moved sinusoidally in the X-direction with an amplitude 5 µm? Express the motion of the sphere under steady-state conditions. (8 pts)

4. A 50mW laser is focused into a Gaussian spot with width w_0=600nm. Assuming that a particle experiences a force proportional to the gradient of the intensity, plot the force along the x-axis in the focal plane (z=0). What is the maximum force? ($n_m = 1.33$, $n_{particle} = 1.6$).

Useful equations:

The gradient force acting on an electric dipole is: $F = \frac{1}{2}\alpha\nabla E^2$ The polarizability of the dipole is:

$$\alpha = n_m^2 R^3 \left(m^2 - 1 \middle/ m^2 + 1 \right), \quad m = n_m \middle/ n_{particle}$$

Intensity of the Gaussian beam is: $I(r,z) = I_0 \left(w_0 \middle/ w(z) \right)^2 \exp\left(-2r^2 \middle/ w^2(r) \right)$

Total power in a Gaussian beam of width w_0: $P_0 = \frac{1}{2}\pi I_0 w_0^2$

BONUS (challenging): Derive the force equation used in prob. 4, starting from the formula for Lorentz force.

Previous lecture was loosely based on Keir Neuman's talk available here:
www.anderswallin.net/sandbox/2005_07_15_neuman_adsl.mp4
Highly recommended listening while you are doing the exercises!