(530190) Methods in Single Molecule Biophysics, Fall 2011

Exercise 1: Wednesday 9.11.2011 at 12:15 in room D116

1.

a) How many water molecules are there in a cup of coffee (volume 2 dl)?

b) Consider a bacterial cell of cylindrical shape, 2 µm long and 500 nm in diameter. The ATP concentration within the cell is 3 mM. How many ATP molecules does the cell contain?

2.

a) A kinesin molecule can generate a steady force of 6 pN. What would be the velocity of a virus (radius 50 nm) that is propelled by the kinesin within the cytoplasm? Cytoplasm has 1000 times higher viscosity than water $\eta(H2O) = 10^{-3}$ Pa s.

b) What is the power generated by the myosin motor as it moves along? If the hydrolysis of one ATP molecule results in 80 pN*nm of energy, how many ATPs must be hydrolyzed per second ?

3.

a) Assume a spherical bacterium of diameter 2 um is swimming along in water at 20°C with a velocity of 25 µm/s. What force must the bacterial motor generate to overcome friction?

b) Assume that the motor suddenly stops working. How long will the bacterium continue to coast after the motor has stopped? Is this time and distance significant? (assume a density of 1.2 times the density of water for the bacterium).

c) Repeat the calculation for a person swimming in water with a velocity of 0.5 m/s. Compare the values with results obtained from a) and b). Hint: approximate the person with a suitably large sphere.

4.

a) show that a time series of random numbers (f1,f2,f3,f4,...) with time-interval dt has
a constant ("white") power-spectral density (PSD) equal to P if the random numbers are
generated from a normal distribution
N(0, sqrt(P/(2*dt)))   i.e. normally distributed random numbers with zero mean and
standard deviation sqrt(P/(2*dt))
- tip1: the matlab function randn() generates N(0,1) numbers
- tip2: the matlab function pwelch() calculates PSD
- tip3: generate many time-series, calculate their power-spectral density and show that
the average power-spectral density converges towards P

b) simulate the diffusion of a r=5 nm protein in water at room-temperature (e.g. for a time of 100ms) :
- How should you choose dt?
- Show that on average diffusion transports the protein nowhere!
- Show that the average squared ("root-mean-square") end-position of the protein is sqrt(2*D*t), where D = kB*T/gamma
- Show that the time-series of position has a 1/f^2 power-spectral density. (bonus: derive the exact PSD and compare to simulation)

c) How justified is leaving out the mass-term from the equation of motion? If we instead solve a system:
m*a + gamma* dx/dt = Fr
and assume our particle has a radius r, with density 1.3 kg/l, at what size r can we start to neglect the mass-term? Is the m=0 approximation for a protein with r=5 nm?
(compare e.g. to r= 5 um, or r= 5mm !)