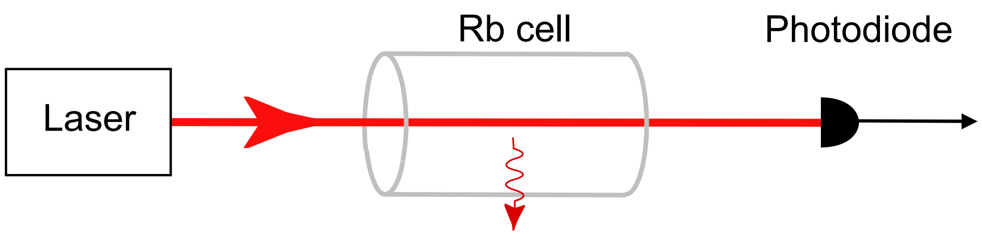
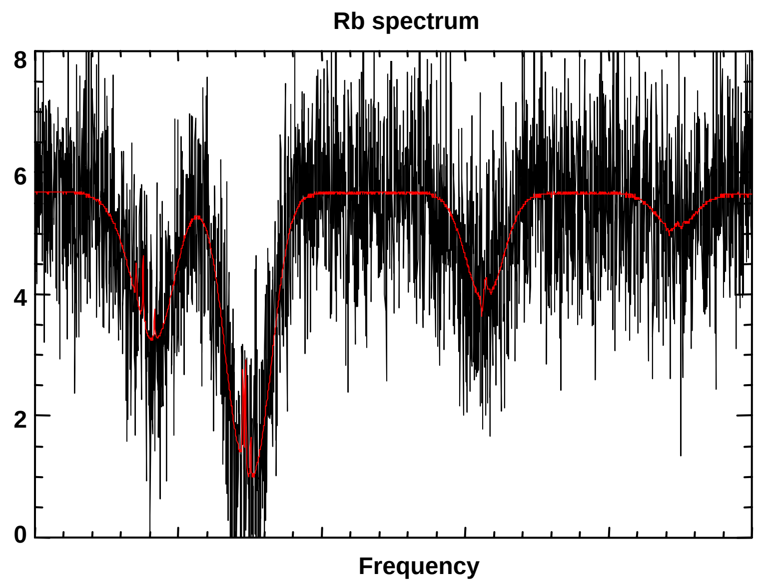
Lock-in amplifiers

A short tutorial by R. Scholten

Measuring something

- Common task: measure light intensity, e.g. absorption spectrum
- Need very low intensity to reduce broadening
- Noise becomes a problem



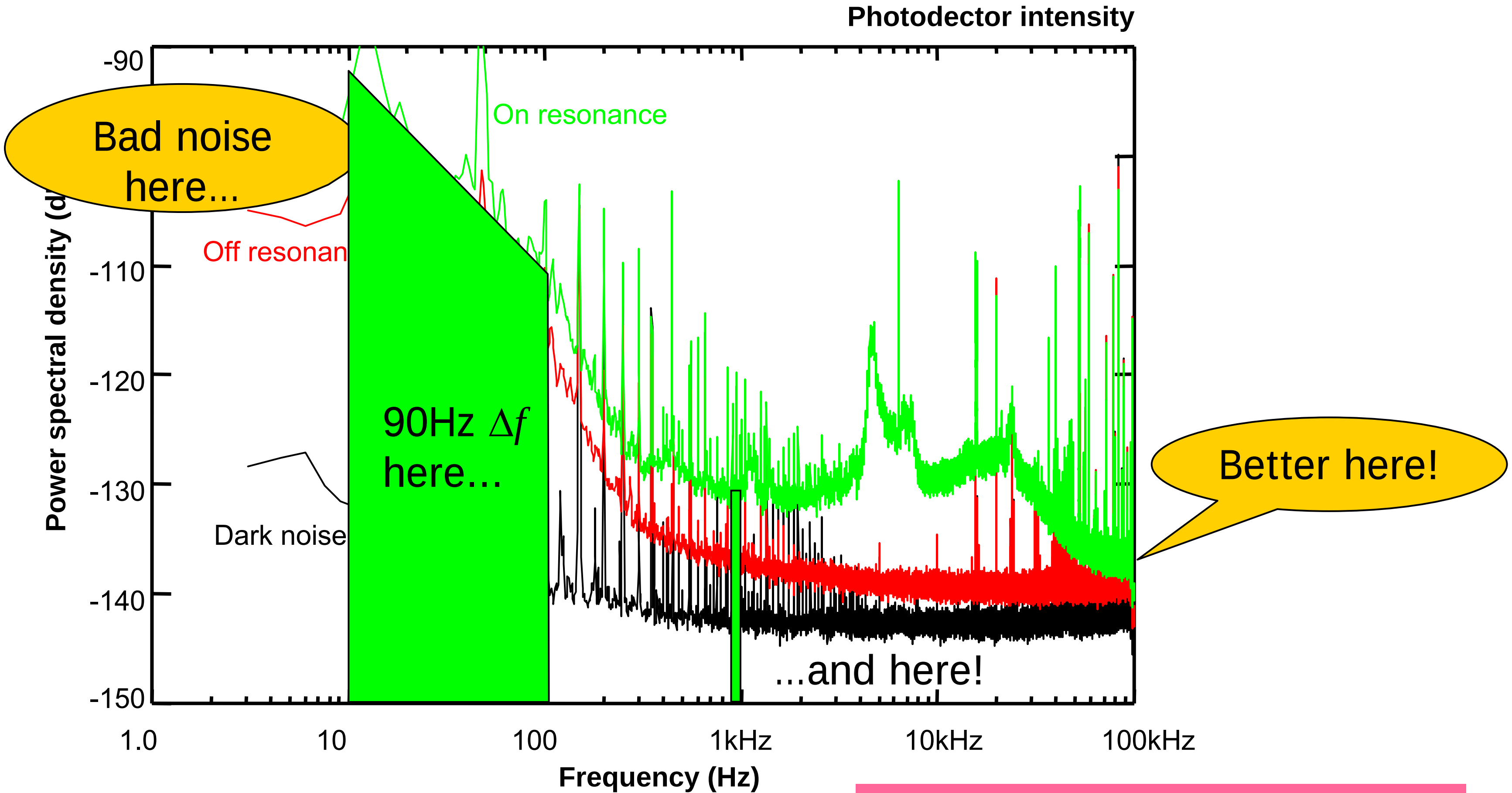


The principle

- Fundamental law of communication theory: ***Wiener-Khinchin*** theorem
- **Reduction** of noise imposed upon a useful signal with frequency f_0, is proportional to the **square root of the bandwidth** of a bandpass filter, centre frequency f_0

Noise

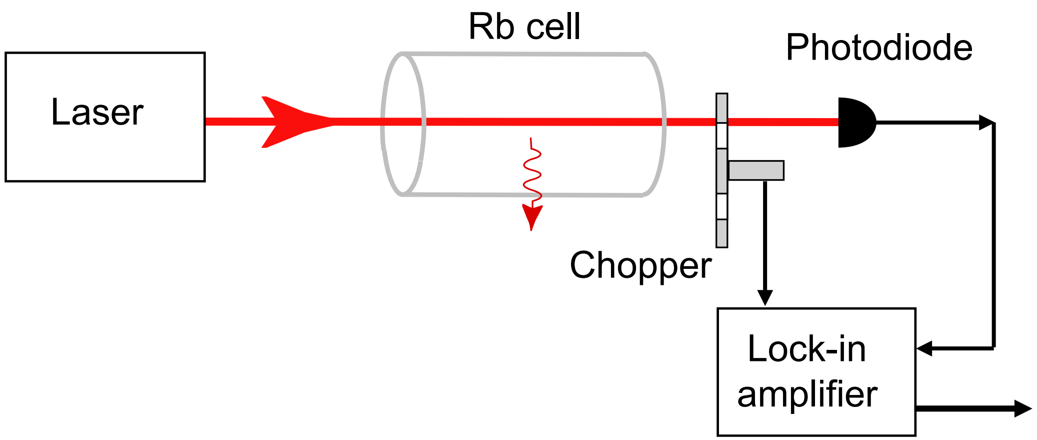
- Typical photodetector noise spectrum

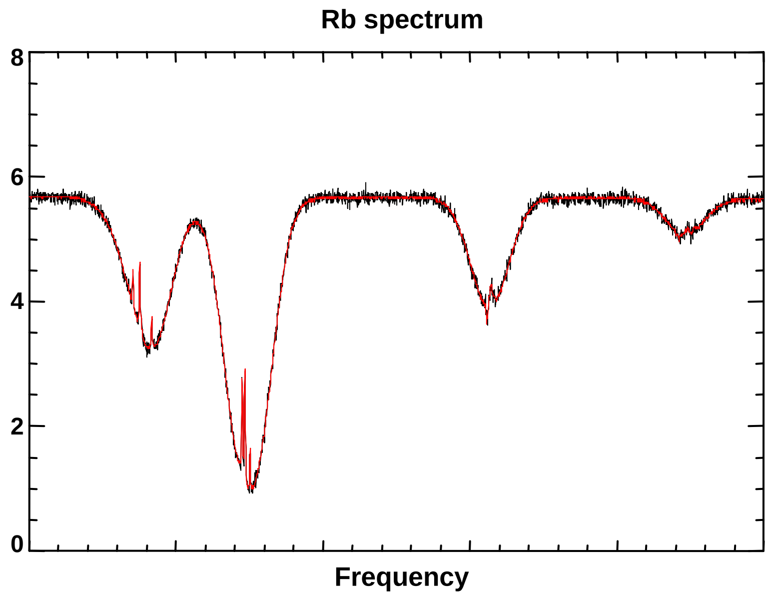


dc measurements:
- broad-spectrum (bad)
- at low frequency (bad)

Measuring something

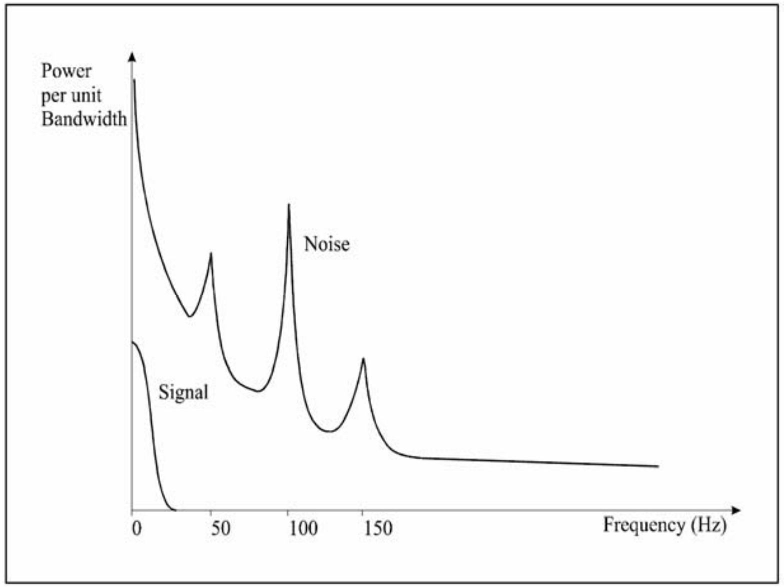
- Need to measure at high frequency, where noise is low
- Modulate signal, look for component oscillating at modulation frequency

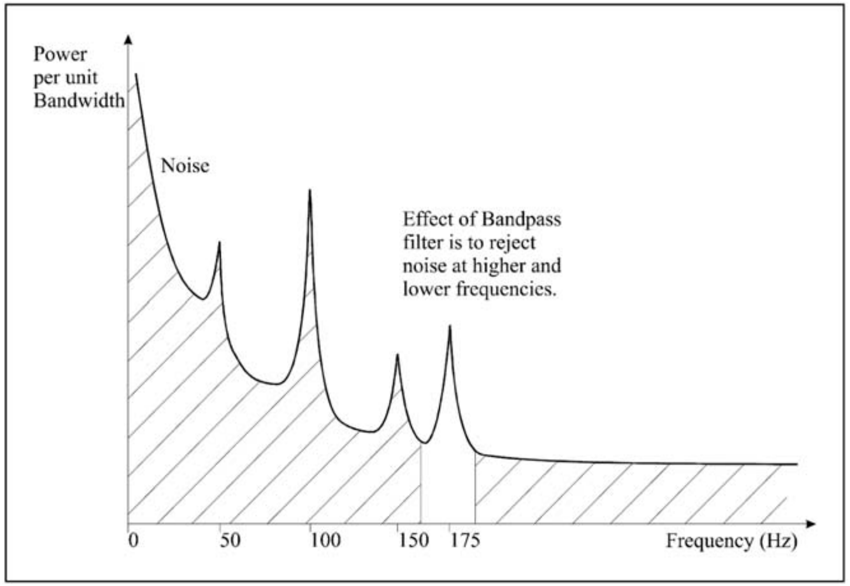




Signal from noise

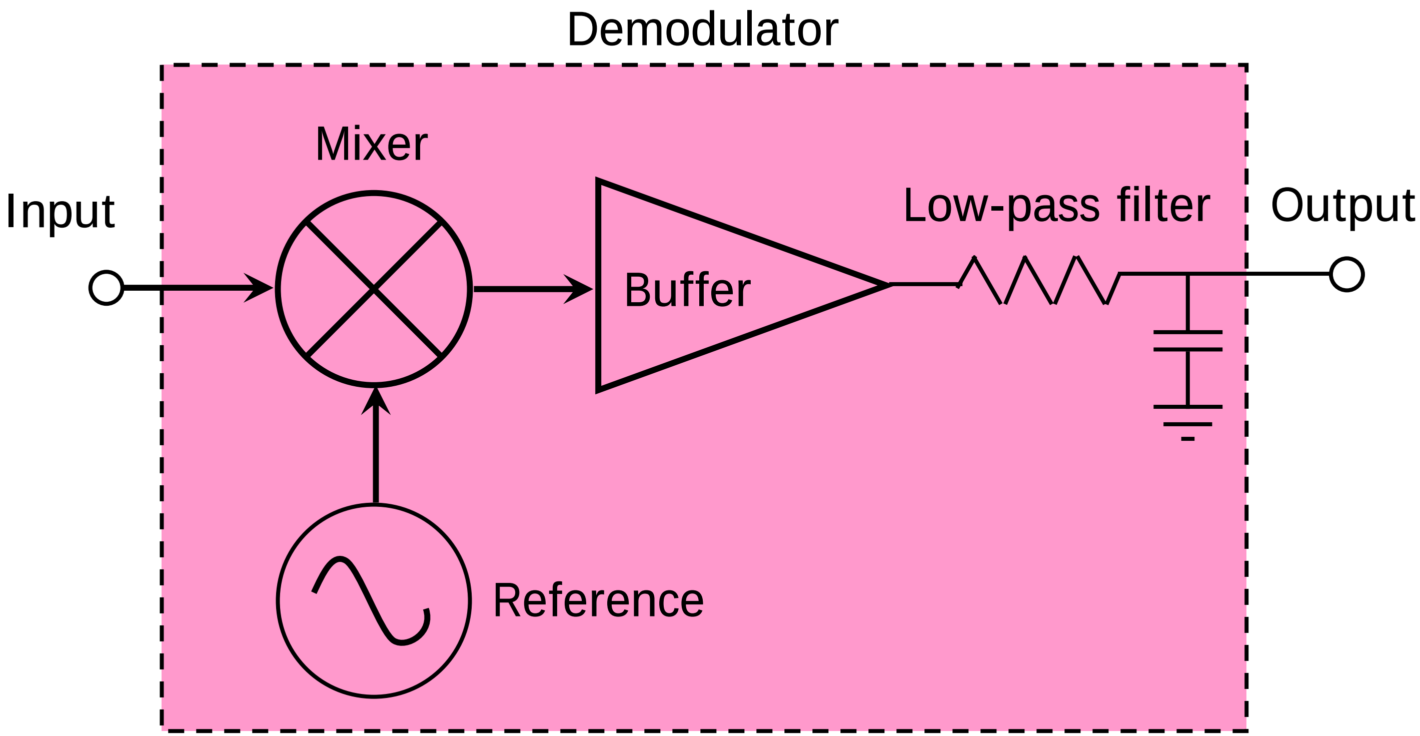
- A *lock-in amplifier* is used to extract signal from noise
- It detects signal based on modulation at some known frequency
- Premise:
  - much noise at low frequency (e.g. dc), less noise at high frequency
  - measure within narrow spectral range, reduce noise bandwidth
- *Hence shift measurement to high frequency*



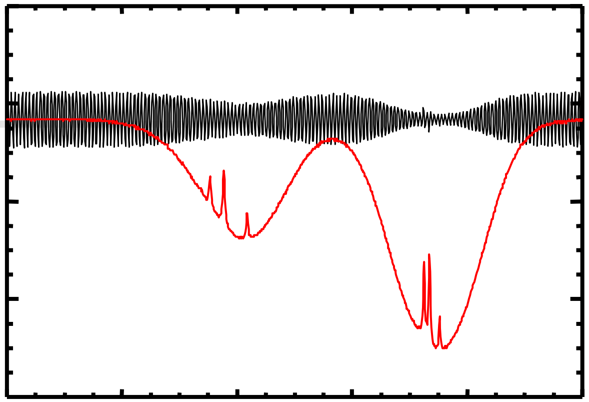


Figures from Bentham Instruments document 225.02 *Lockin amplifiers*

Demodulator or PSD (phase-sensitive detector)



Mathematical description



- Signal $V_S(t)$ varies relatively slowly
  e.g. absorption spectrum scan over 10 seconds

- Modulate at relatively high frequency ω (e.g. chopper): $V_{sig} = V_S(t)\cos\omega t$
- Reference (local oscillator) of fixed amplitude: $V_{ref} = \cos(\omega t + \phi)$
  - phase ϕ is variable
  - oscillator frequency ω same as modulation frequency
- Multiply modulated signal by REF :

$$V_{sig}V_{ref} = V_S(t)\cos\omega t\cos(\omega t + \phi)$$
$$= \tfrac{1}{2}V_S(t)\cos\phi + \tfrac{1}{2}V_S(t)\cos(2\omega t + \phi)$$

- Second term at high frequency (2ω)
- Low-pass filter (cutoff ~ $\omega/2$ or lower)

$$V_{sig} \times V_{ref} \times \text{filter} = \tfrac{1}{2}V_S(t)\cos\phi$$

Note phase-sensitive detection!

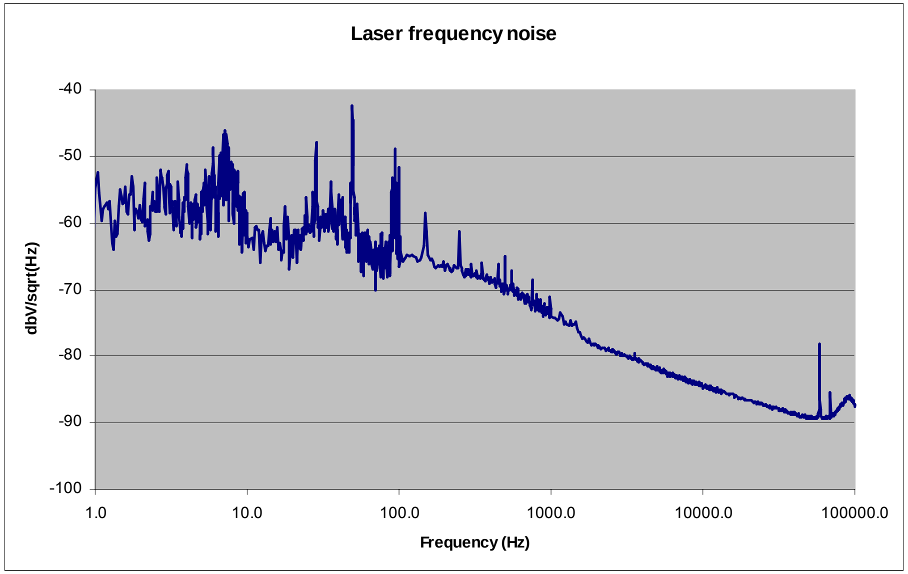
Some details

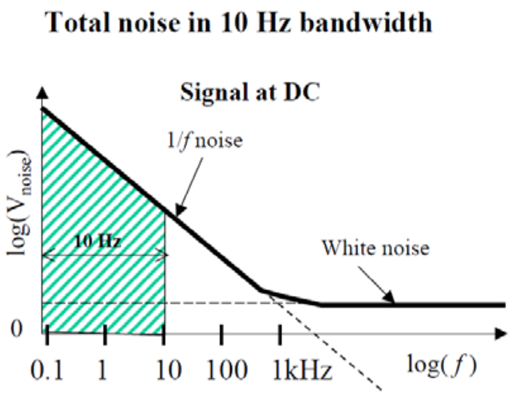
- Simple trig

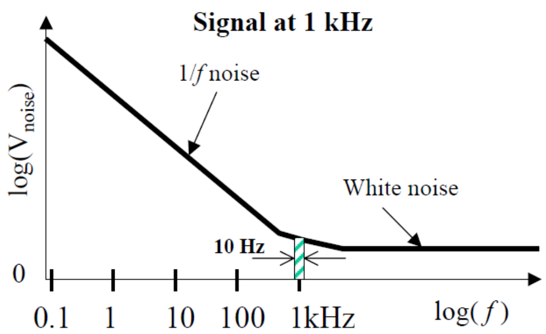
$$
\begin{aligned}
V_{\mathrm{ref}}V_{\mathrm{sig}} &= V_S(t)\cos\omega t\cos(\omega t+\phi)+n(t)\cos(\omega t+\phi) \\
&= V_S(t)\cos\omega t[\cos\omega t\cos\phi-\sin\omega t\sin\phi]+\ldots \\
&= V_S(t)[\cos^2\omega t\cos\phi-\cos\omega t\sin\omega t\sin\phi]+\ldots \\
&= V_S(t)[(½+½\cos 2\omega t)\cos\phi-½\sin 2\omega t\sin\phi]+\ldots \\
&= ½V_S(t)[\cos\phi+\cos 2\omega t\cos\phi-\sin 2\omega t\sin\phi]+\ldots \\
&= ½V_S(t)\cos\phi+½V_S(t)[\cos 2\omega t\cos\phi-\sin 2\omega t\sin\phi]+\ldots \\
&= \tfrac{1}{2}V_S(t)\cos\phi+\tfrac{1}{2}V_S(t)\cos(2\omega t+\phi)+n(t)\cos(\omega t+\phi)
\end{aligned}
$$

Noise

- Noise reduces with frequency ($1/f$ noise is major problem)
- Shift signal to higher frequency
- Noise within given bandwidth reduces as we measure at higher frequency







www.technion.ac.il/technion/chemistry/courses/w05/127421

With noise

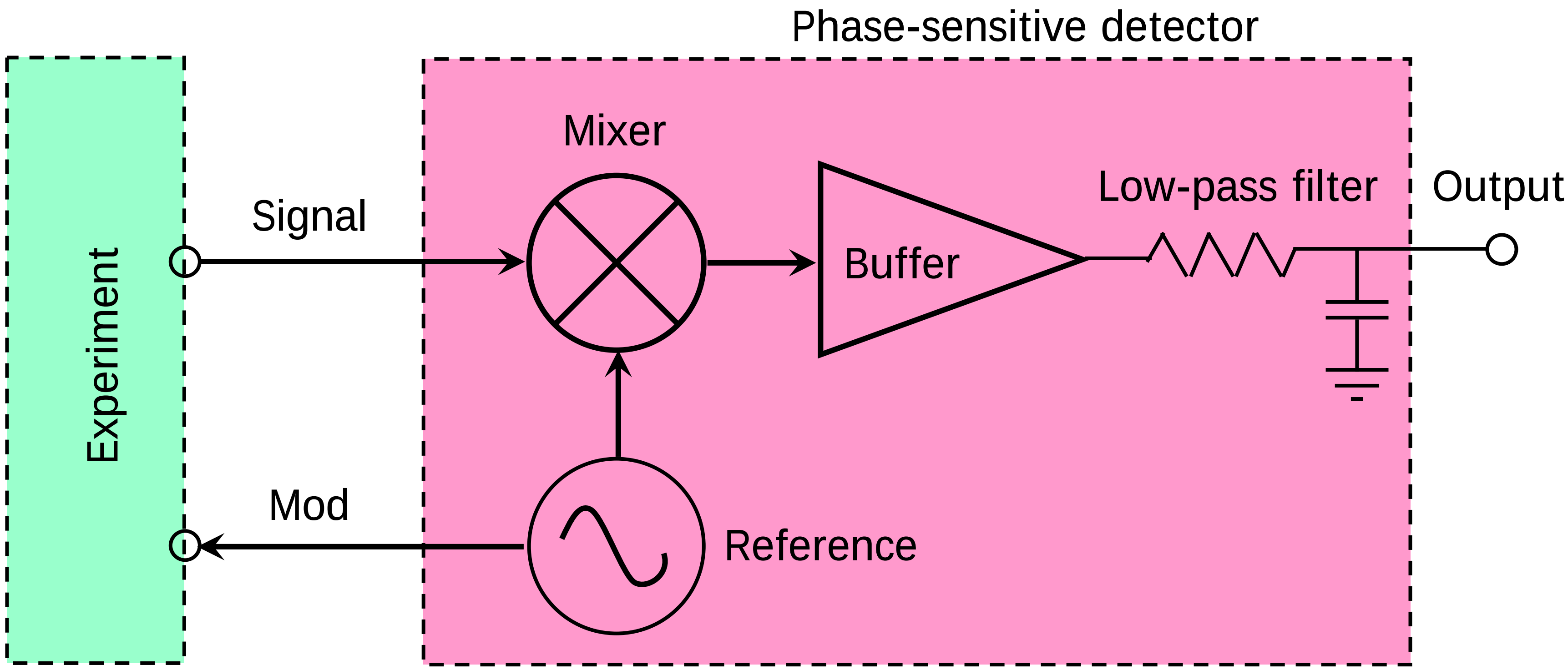
- Signal has noise: $V_{\text{sig}} = V_S(t)\cos\omega t + n(t)$

- Multiply reference by modulated signal:

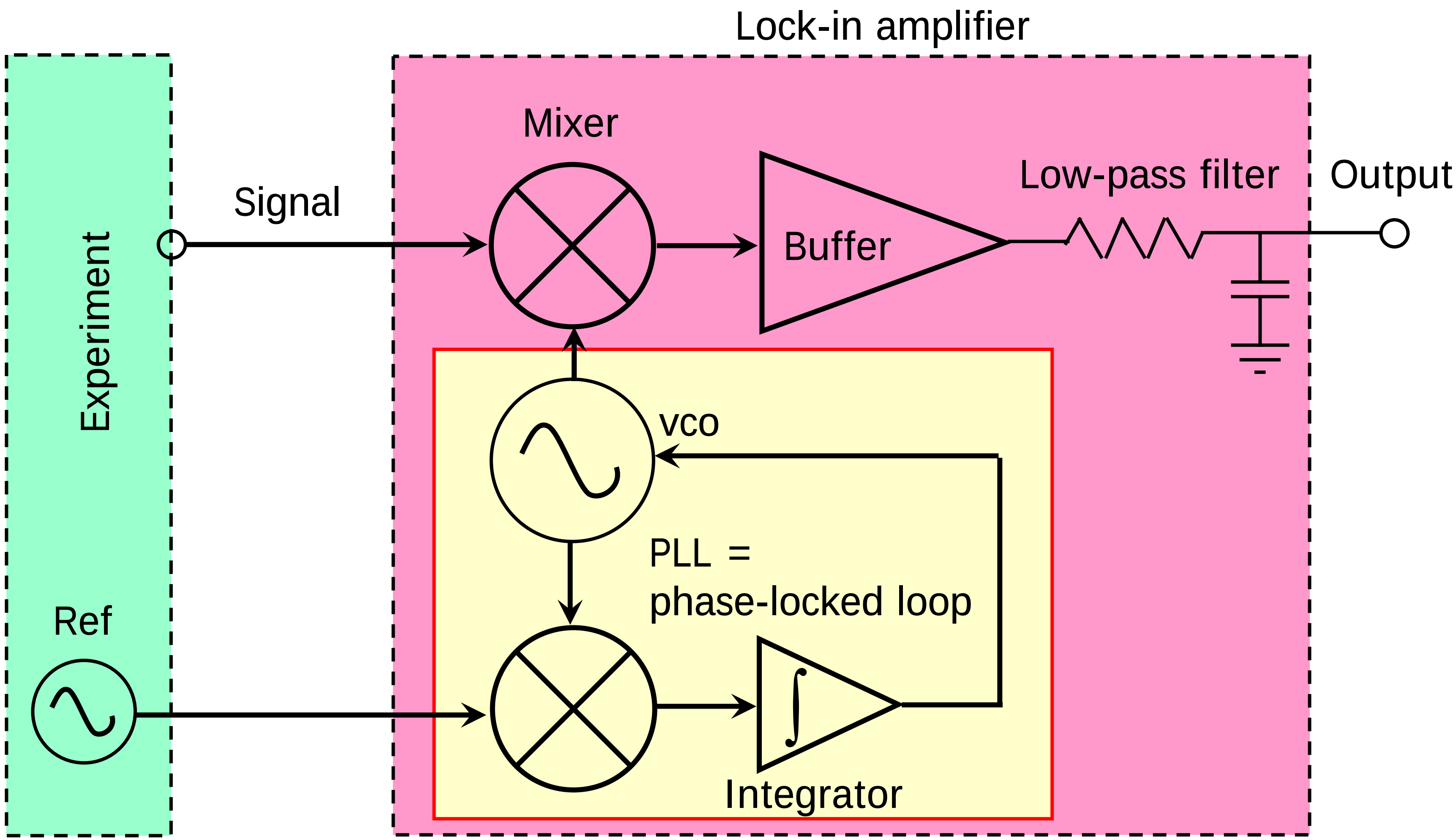
$$\begin{aligned} V_{\text{ref}}V_{\text{sig}} &= V_S(t)\cos\omega t\cos(\omega t+\phi) + n(t)\cos(\omega t+\phi) \\ &= \tfrac{1}{2}V_S(t)\cos\phi + \tfrac{1}{2}V_S(t)\cos(2\omega t+\phi) + \underbrace{n(t)\cos(\omega t+\phi)} \end{aligned}$$

- Third term – noise – at frequency ω
- Low-pass filter, frequency less than $\omega/2$, leaves signal components
- We win twice:
  - less noise at ω
  - reduce bandwidth

Using PSD oscillator to modulate



External modulator: true “lock-in”



Further details

- True lock-in amp can work with external oscillator for Reference:
  - Input reference from external experiment
  - Use phase-locked-loop to generate stable local oscillator

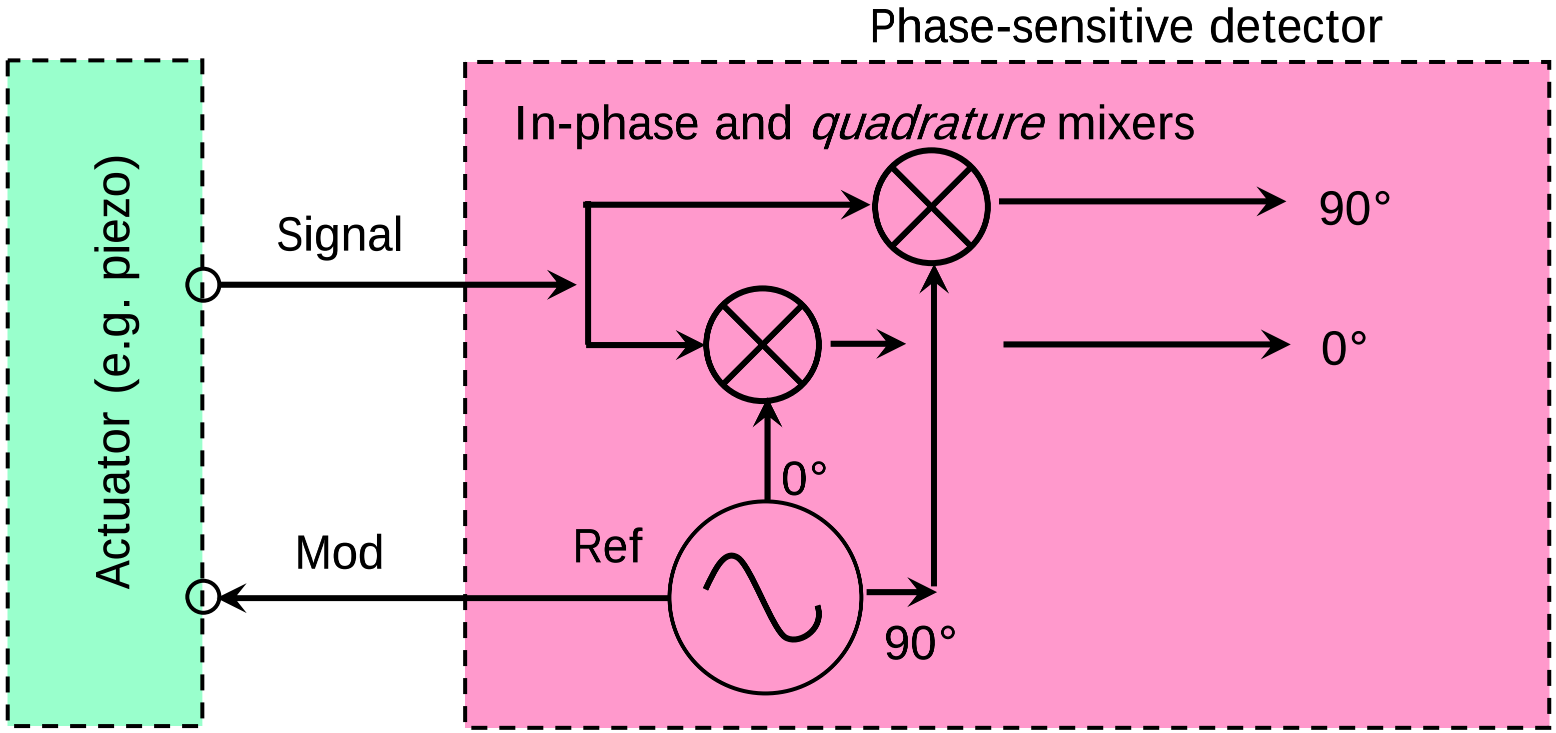
- Lock-in amp has variable post-multiplier (low-pass) filter
  - Time constants: what time constant is appropriate?
  - Shapes (6^{th}, 12^{th}, ... order): which is best?

- If input signal has ***harmonics*** (e.g. due to imperfect modulation) then will detect spurious signal
  - Use ***input*** filter to minimise

- Dynamic reserve?

Other applications

- Often use lockin to measure response function of actuator (or similar)
- Two-channel lockin – measure signal and *phase*
- Phase → resonances



Experiments

- Photodiode + LED
- SRS FFT spectrum analyser
- Oscilloscope
- Switch LED on/off, e.g. with hand to block
- HP function generator to modulate LED
- And/or chopper
- SRS lock-in amp

The other half of the story

Frequency modulation

Derivatives: Lock-in amps & feedback servos

- So far, we have modulated *amplitude,* and used LIA to demodulate
- PSD (lockin) = fancy bandpass filter?!

- Can also use *frequency* modulation (like FM radio)

- Let's measure $V_S(\omega)$ i.e. a spectrum, where we slowly vary $\omega(t)$

- Frequency-modulate: $\omega(t) = \omega_0 + \Omega_0 \cos \Omega t$
  where Ω is the modulation (Fourier) frequency

- Using Taylor-series expansion: $V_S(\omega) = V_S(\omega_0) + \left.\frac{dV_S}{d\omega}\right|_{\omega=\omega_0} (\Omega_0 \cos \Omega t) + \ldots$

- Note two things immediately:
  - dc component is same as un-modulated spectrum
  - ac component is *proportional to derivative* of spectrum

Extract *derivative* with PSD/lock-in amp

- We now multiply our signal by our reference, as before:

$$V_{\text{sig}} = V_S(\omega_0) + \left.\frac{dV_S}{d\omega}\right|_{\omega=\omega_0} (\Omega_0 \cos\Omega t) + \ldots$$

$$V_{\text{ref}} = \cos(\Omega t + \phi)$$

- Note *modulation* at Ω, and fixed ω_0 i.e. slowly varying laser frequency

$$\begin{aligned} V_{\text{sig}} V_{\text{ref}} &= V_S \cos(\Omega t + \phi) + \frac{dV_S}{d\omega} \Omega_0 \cos\Omega t \cos(\Omega t + \phi) \\ &= \ldots \\ &= V_S \cos(\Omega t + \phi) + \tfrac{1}{2}\Omega_0 \frac{dV_S}{d\omega} \cos(2\Omega t + \phi) + \tfrac{1}{2}\Omega_0 \frac{dV_S}{d\omega} \cos\phi \end{aligned}$$

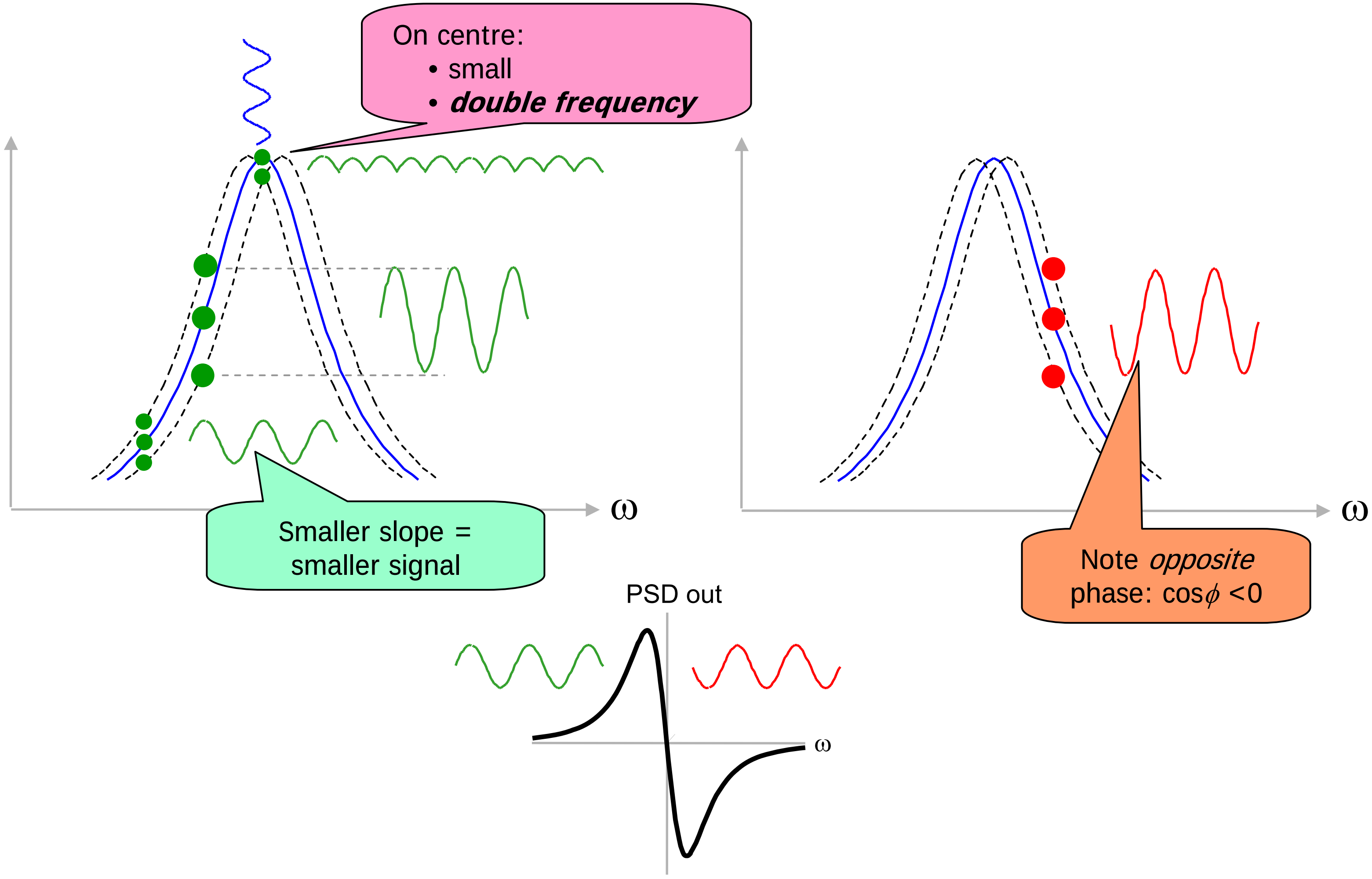
- Again: low-pass filter (cutoff ~ $\omega/2$ or lower)

$$V_{\text{sig}} V_{\text{ref}} \approx \tfrac{1}{2}\Omega_0 \frac{dV_S}{d\omega} \cos\phi$$

- We have a measurement proportional to the *derivative*
- Measurement *changes sign* if slope changes sign: dispersion
- Note: modulation depth Ω_0 must not be larger than peak in spectrum!
- Higher-order terms in Taylor expansion: can measure 2nd deriv, 3rd deriv, etc.

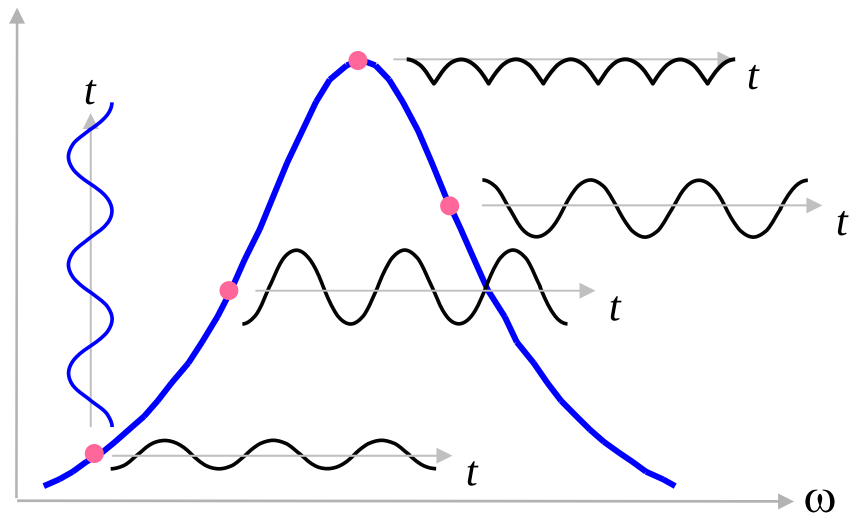
Lock-in amplifiers and feedback servos

- Example: Lorentzian peak in atomic absorption spectrum

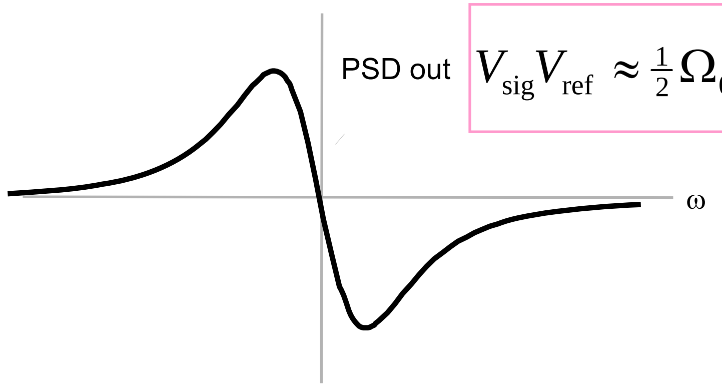


Lock-in amps in servos

- Modulated output from detector



- Demodulated output from lock-in



$$V_{\text{sig}} V_{\text{ref}} \approx \tfrac{1}{2}\Omega_0 \frac{dV_S}{d\omega} \cos\phi$$

Yet other half of the story

Spread-spectrum

PRBS: Lock-in on steroids!

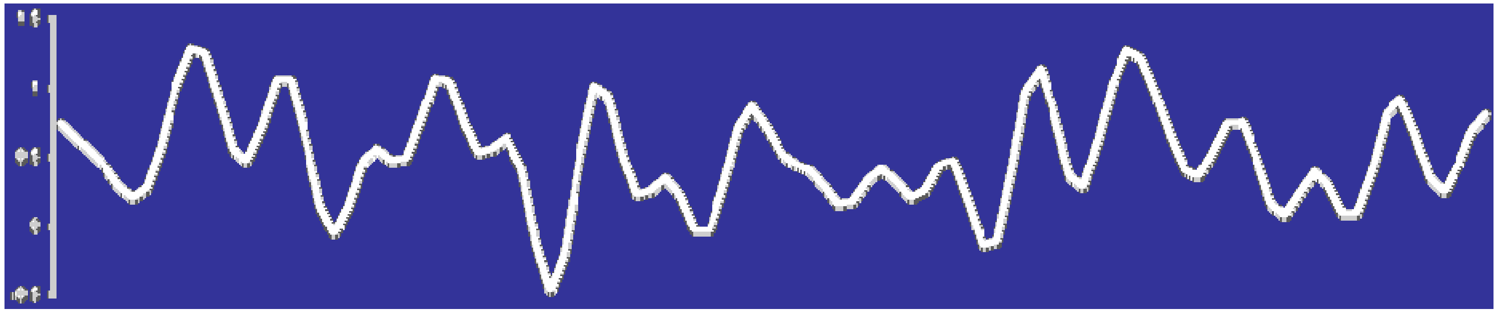
- Lock-in uses small part of spectrum
- Can use broad spectrum and still separate signal from noise
- Pseudo-random bit sequence
  - Spread-spectrum communications
    - computer 802.11 wireless, etc.
    - CDMA telephones
    - Modems
  - Security/encryption
  - Acoustics

Spread-spectrum modulation

SPREAD-SPECTRUM MODULATION

- all frequencies present simultaneously in modulation function
- phases adjusted so that components add *in quadrature*

http://www.chm.bris.ac.uk/pt/mcinet/sum_schl_02_docs/tof.ppt



20 frequencies
random phases

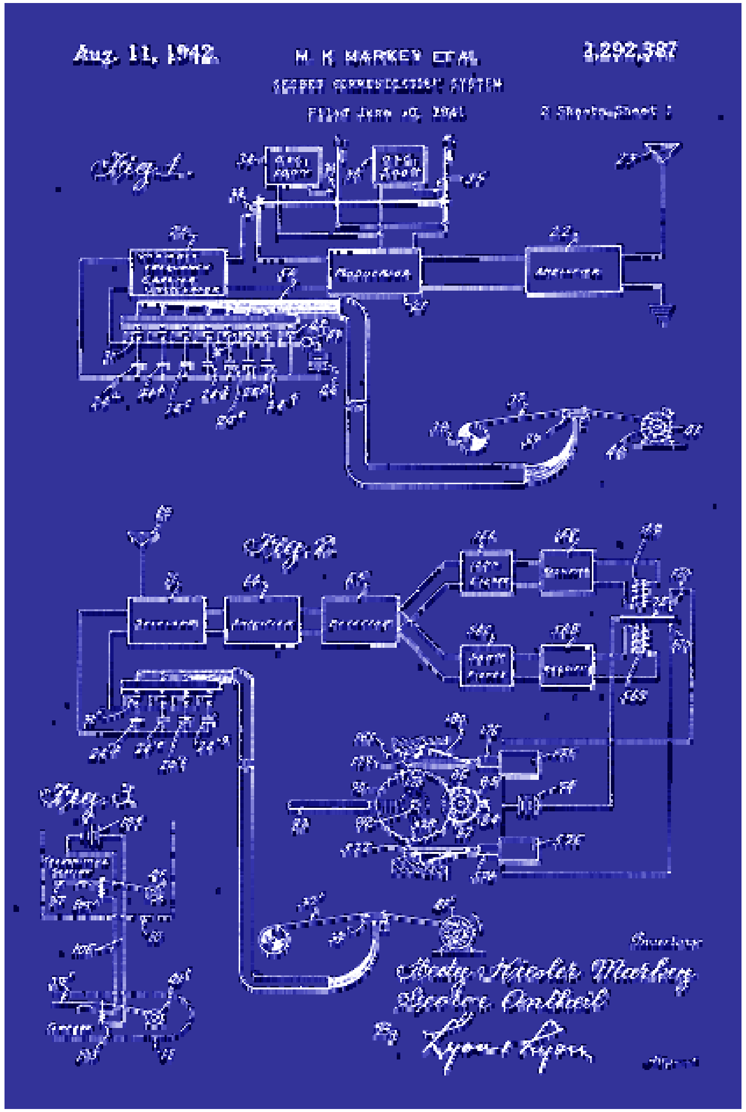
- truly random phases cause excursions out of range
  $\Rightarrow$ use *pseudo-random* functions

http://www.chm.bris.ac.uk/pt/mcinet/sum_schl_02_docs/tof.ppt

Spread-spectrum history

Also famous as first nude in cinema-release movie!

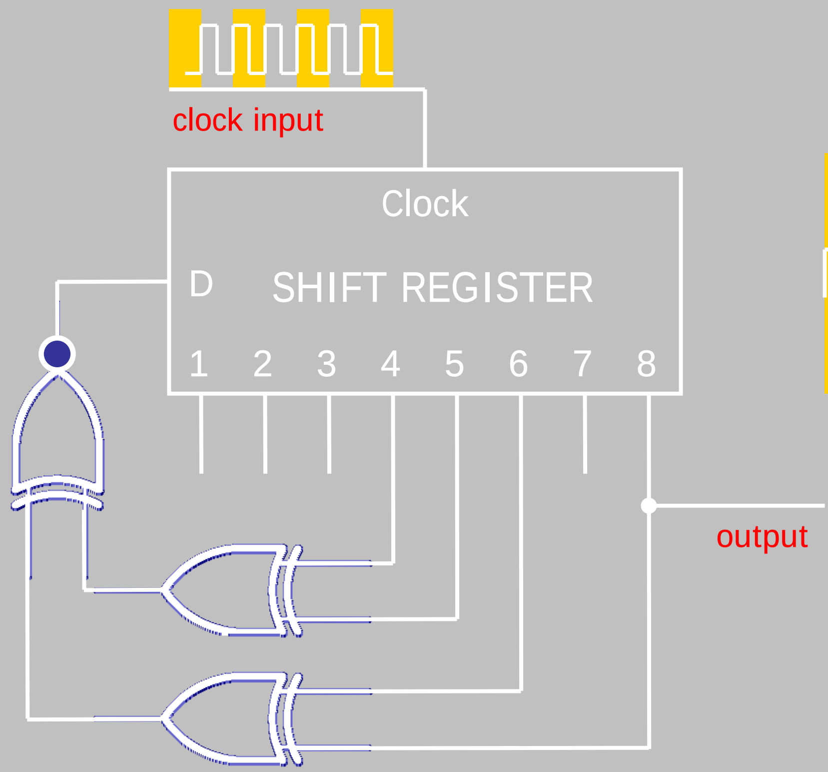
- Hedy Lamarr (1913-2000), composer George Antheil (1900-1959) patented submarine communication device
- Synchronized frequency hopping to evade jamming
- Original mechanical action based upon pianolas
- Used today in GPS, cellphones, digital radio

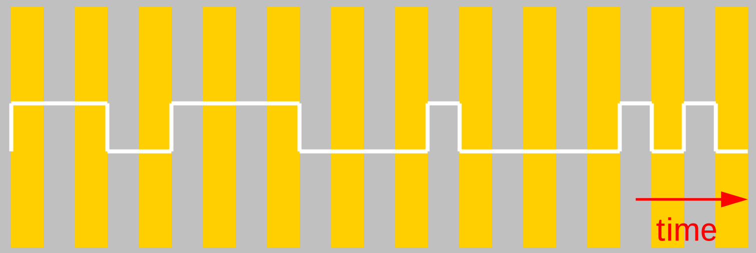


http://www.ncafe.com/chris/pat2/index.html

http://www.chm.bris.ac.uk/pt/mcinet/sum_schl_02_docs/tof.ppt

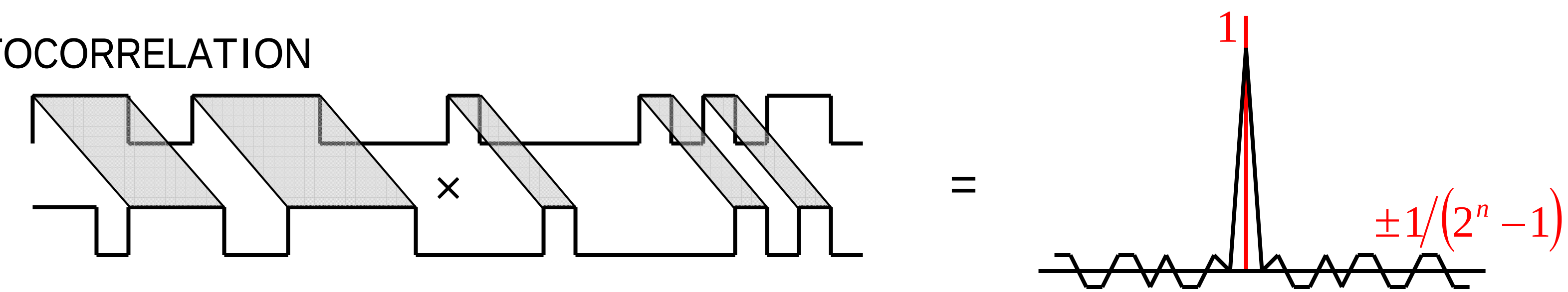
Binary pseudo-random sequences





- sequence length $2^n - 1$ bits with n bit shift register

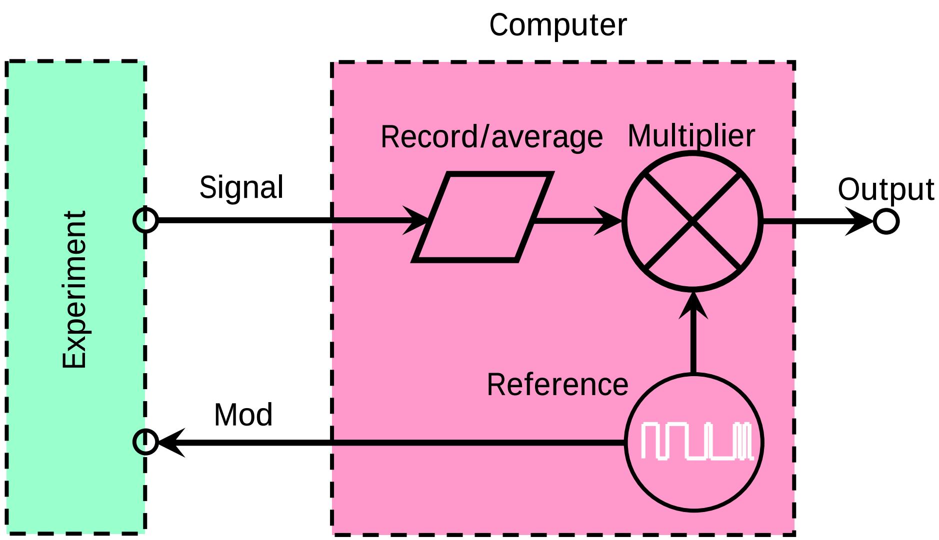
AUTOCORRELATION



http://www.chm.bris.ac.uk/pt/mcinet/sum_schl_02_docs/tof.ppt

PRBS: Lock-in on steroids!

- Generate signal in pseudo-random bit sequence, for example:
  - 6-bit (64-bits long)
    - 0110101011111100000100001100010100111101000111001001011011110110
  - 8-bit (256 bits long):
    - 00011011011100100010110110010110011101101011101010011011110011111010110001010000111101001
      11100010001110100010011001001110011010101101001010111111011101111100000110011000010000101
      010100011000111111110010100100000001011110110000001110001101000000100100101110
- Record signal
- Multiply by PRBS (auto-correlate)
- Very much like a lock-in! But uses broad spectrum



13-bit (8192 bits long) MLS single scan